GPU-accelerated integral equation method for 3D modelling of induction logs

D.H. Saputera[1], M. Jakobsen[1], N. Jahani[2], S. Alyaev[2], K.S. Eikrem[2], K.W.A. van Dongen[3]

[1]Department of Earth Science, University of Bergen, Bergen, Norway

[2]NORCE Norwegian Research Centre, Bergen, Norway

[3]Department of Imaging Physics, Delft University of Technology, Delft, The Netherlands

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Motivation

The integral equation method (IE) is commonly used to model controlled-source electromagnetic data, but there are only a few successful IE applications to 3D modelling of induction logs. Efficient implementations of the IE are typically based on a combination of Krylov subspace methods for linear systems iterative solver and the FFT algorithm for efficient convolution of the dyadic Green's function with various contrast-source terms. If the contrasts are large and/or the anisotropy is strong, the FFT-Krylov based IE method often requires a large number of iterations. In this study, we reduce the number of iterations required to solve the IE to a given tolerance by using a contraction integral preconditioner. By implementing the contraction FFT-Krylov method using GPU as well as CPU, we have made significant progress towards the possibility of performing real-time 3D modelling of induction logs. Our numerical examples also include an encouraging comparison with analytical results for a homogeneous model and finite difference (FD) simulations for an anisotropic layered model.

Integral equation method

Electric fields in the integral equation form:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E_b}(\mathbf{r}) + \int_V \bar{\bar{\mathbf{G}}}^{\mathbf{E}}(\mathbf{r},\mathbf{r}')\Delta\bar{\bar{\sigma}}(\mathbf{r}')\mathbf{E}(\mathbf{r}')\,dr'^3$$

In the discretized form:

$$\mathbf{AE} = \mathbf{E_b}$$

$$\mathbf{A} = \bar{\bar{\mathbf{I}}} - \bar{\bar{\mathbf{G}}}^{\mathbf{E}}\Delta\bar{\bar{\sigma}}$$

This linear system can be solved using Krylov subspace iterative method.

Pre-conditioner is applied to ensure the convergence of the iterative solvers and faster convergence [3]:

$$\mathbf{M_1 A M_2}(\mathbf{M_2^{-1}E}) = \mathbf{M_1 E_b}$$

$$\mathbf{M_1} = \sqrt{Re\sigma_b}$$

$$\mathbf{M_2} = 2\sqrt{Re\sigma_b}(2Re\sigma_b\bar{\bar{\mathbf{I}}} + \bar{\bar{\sigma}})^{-1}$$

which can be rewritten as:

$$\tilde{\mathbf{A}}\tilde{\mathbf{E}} = \hat{\mathbf{E}}_\mathbf{b}$$

The iterative solver will converge since $\tilde{\mathbf{A}}$ is a contraction operator.

The convolution integral of the dyadic Green's function can be efficiently calculated using FFT [1]:

$$\bar{\bar{\mathbf{G}}}^{\mathbf{E}}\Delta\bar{\bar{\sigma}}\mathbf{E} = \mathcal{F}^{-1}(\mathcal{F}(\bar{\bar{\mathbf{G}}}^{\mathbf{E}}) \odot \mathcal{F}(\Delta\bar{\bar{\sigma}}\mathbf{E}))$$

After solving the integral equation for the electric fields, the magnetic fields can be calculated:

$$\mathbf{H} = \mathbf{H_b} + \bar{\bar{\mathbf{G}}}^{\mathbf{H}}\Delta\bar{\bar{\sigma}}\mathbf{E}$$

$$\bar{\bar{\mathbf{G}}}^{\mathbf{H}} = (i\omega\mu_0)^{-1}\nabla \times \bar{\bar{\mathbf{G}}}^{\mathbf{E}}$$

Mathematical operations with high degree of parallelization can be computed more efficiently by using GPU due to the high number of cores [4].

Acknowledgements

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IE Accuracy and Speed for homogeneous isotropic model

We specify σ_b= 1 S/m and $\Delta\sigma$= 0.1 S/m to calculate the magnetic fields in a homogeneous isotropic medium using IE with σ= 1.1 S/m and source at the origin with 12 kHz frequency.

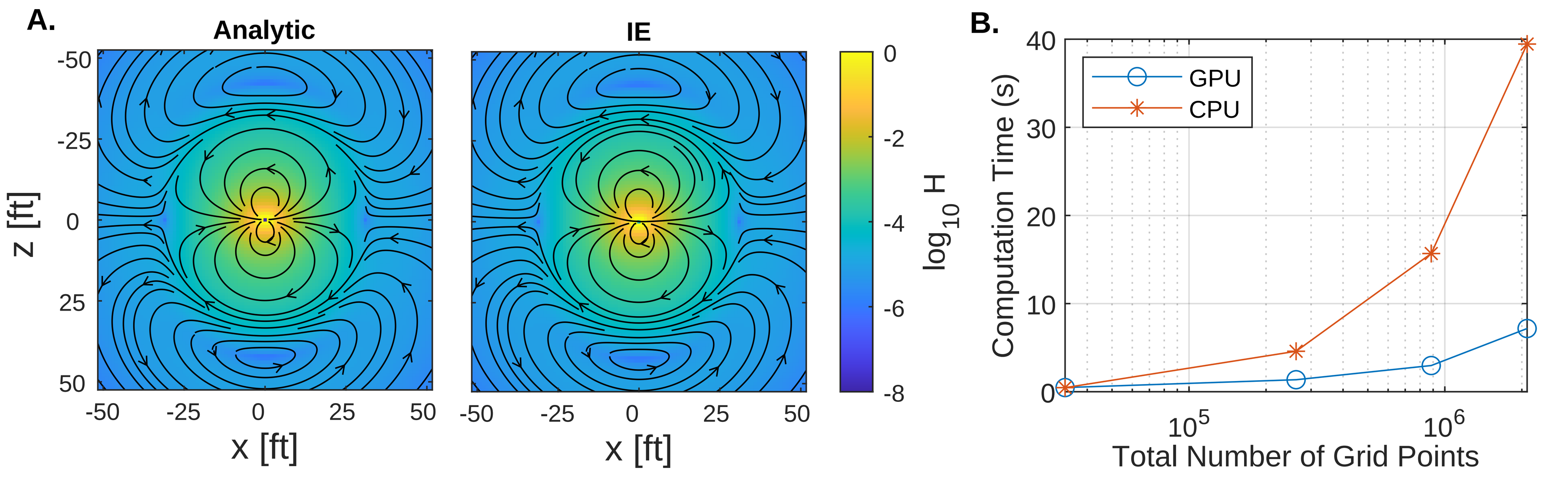

Fig. 1.A. Field lines and magnitude of magnetic fields real component calculated using analytical solution and IE. **B.** Comparison between GPU and CPU computation time for Homogen Isotropic case.

Fig.1.A. shows a good match between analytical solution and IE modelling result with less than 1% magnitude difference. The IE code is executed on a laptop with AMD Ryzen 7 4800H processor and NVIDIA GeForce RTX 3060 Laptop GPU. As shown in **Fig.1.B.**, the computational savings when using GPU increases proportionally with the total number of simulation grid points.

Comparison of IE and FD for anisotropic layered model

Each point along the drilling trajectory correspond to one IE simulation domain with source at the origin. The simulation domain axes are rotated according to the drilling angle. The source frequency is 12 kHz and a receiver is located 25 ft away from the source.

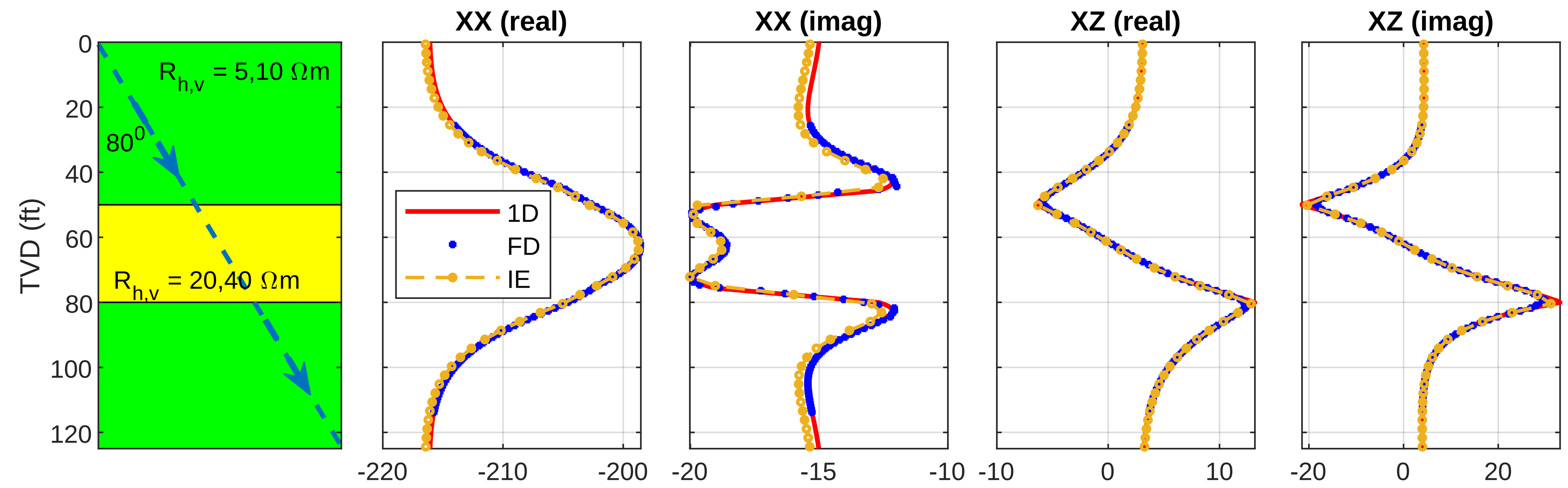

Fig. 2. XX and XZ component along the drilling trajectory indicated by blue dashed line in the model. All fields are in μA/m.

The calculated magnetic fields using IE show similar trend with the finite difference [2] and 1D analytical solution. Components with X-Y and Z-Y coupling are zero. Calculating 100 logging positions with 128 x 128 x 128 simulation grid using IE took approximately 1 hour 48 minutes.

3D Modelling of induction logs in complex media

We simulated induction logs across faulted anisotropic formation using IE. The drilling angle is 85 degrees, source frequency is 48 kHz and source-receiver spacing is 25 ft.

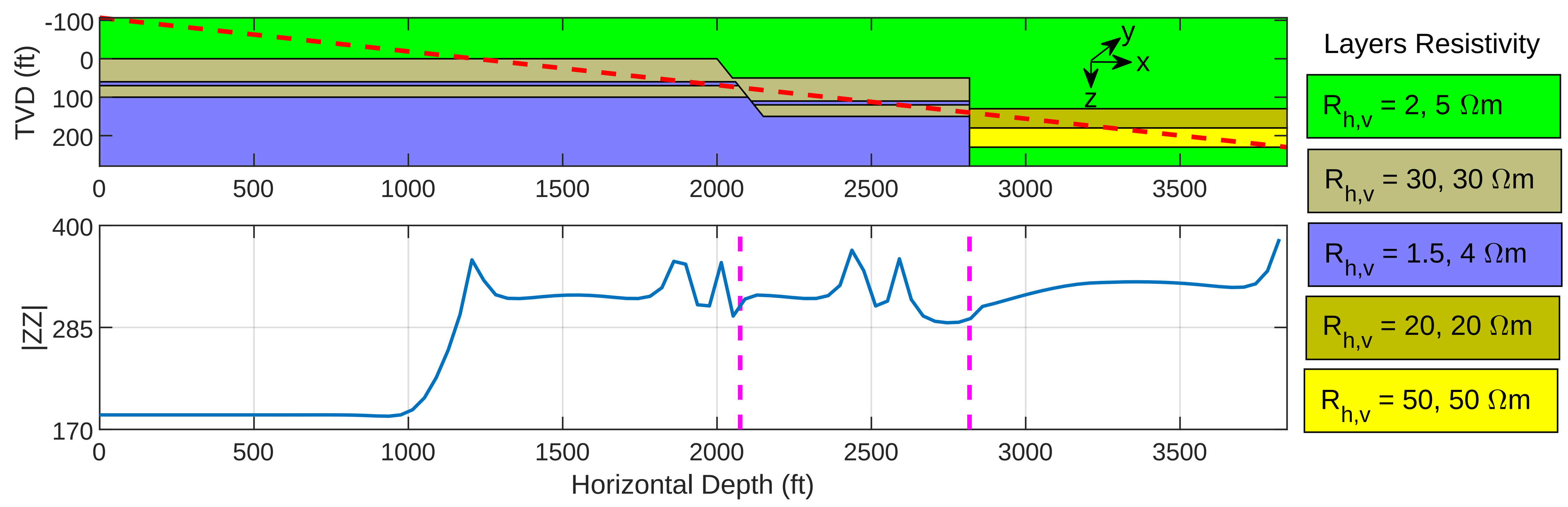

Fig. 3. The magnitude of ZZ component simulated along the drilling trajectory indicated by red dashed line in the model. Magenta lines indicate the intersection of drilling trajectory with the faults.

The total computation time is approximately 2 hours 48 minutes for 100 logging positions with 128 x 128 x 128 simulation domain. Spiky shapes in the |ZZ| curve indicate layer interfaces with large contrast.

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